

Using branching processes in nuclei to reveal dynamics of large-angle, two-body scattering

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Abstract

We demonstrate that hard branching $2 \rightarrow 3$ particle processes with nuclei provide an effective way to determine the momentum transfers needed for effects of point-like configurations to dominate large angle $2 \rightarrow 2$ processes. In contrast with previously proposed approaches, the discussed reaction allows the effects of the transverse size of configurations to be decoupled from effects of the space-time evolution of these configurations.

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Large angle two body processes seem to be amongst the simplest hadronic interactions that can be described by perturbative quantum chromodynamics (pQCD). In the limit of $s \rightarrow \infty, t/s = \text{const}$ these processes are dominated by hard gluon exchanges between the constituents with all of the quark constituents at small relative distances [1, 2]. Contributions of large size configurations (in particular the end point contribution) should be suppressed in this limit by Sudakov form factors, see discussions in Refs. [3, 4]. Nevertheless, after many years and many studies the momentum transfer range of applicability of pQCD has not been firmly established. A series of experiments at Brookhaven National Laboratory (BNL) measured the nuclear transparency of nuclei in quasielastic scattering process near 90° in the pp center of mass, see the summary in [5]. An increase of nuclear transparency was observed between $p_N^{\text{inc}} = 5.9 \text{ GeV}/c$ and $p_N^{\text{inc}} = 9.5 \text{ GeV}/c$, indicating that freezing of nucleonic configurations becomes possible for $p_N \geq 8 \text{ GeV}/c$. This rise is followed by drop of the transparency at larger incident momenta indicating that some nonperturbative mechanisms play an important role in nucleon-nucleon scattering (for the reviews see e.g. [6, 7]) up to the momentum transfer squared $-t \sim 13 \text{ GeV}^2$. At the same time, the observed enhancement of the K^+p elastic cross section as compared to π^+p elastic cross section for $\theta_{c.m.} = 90^\circ$ and $-t \geq 5 \text{ GeV}^2$ suggests that scattering processes are dominated by point-like configurations (PLC) in mesons that have a larger probability for the K^+ -meson than for the pion (see discussions in the beginning of Sec.II and also in the end of Sec.VI of Ref. [8]). In principle, the onset of the regime of dominance of PLC could be quite different for meson and baryon projectiles because a meson is a much simpler object than a baryon. A larger PLC probability is obtained for the simple reason that only two quarks have a close encounter. In addition, the nonperturbative structure of baryons is likely to be much more complicated. This is indicated in particular by the structure of hadrons in the large N_c limit in which meson remains a $q\bar{q}$ system while a nucleon can be viewed as a soliton [9]. It was nearly three decades ago when Refs. [10, 11] suggested testing the mechanism of these reactions using the color cancellation (CC) property of color-neutral objects of QCD - the suppression of the interaction of small size color singlet wave packets with hadrons. CC plays a key role in ensuring approximate Bjorken scaling in deep inelastic scattering [12], in proving QCD factorization theorems for high energy hard exclusive processes [14], etc. It also leads to color transparency CT under certain conditions (see discussion below). CC may be visualized in the high energy limit by introducing a notion of the scattering cross

section of a small dipole configuration (say $q\bar{q}$) with transverse size d on the nucleon [15, 16] which in the leading log approximation is given by Refs. [17, 18]

$$\sigma(d, x) = \frac{\pi^2}{3} \alpha_s(Q_{eff}^2) d^2 \left[x G_N(x, Q_{eff}^2) + \frac{2}{3} x S_N(x, Q_{eff}^2) \right], \quad (1)$$

where $Q_{eff}^2 = \lambda/d^2$, $\lambda = 4 \div 10$ [19], G_N is the nucleon gluon distribution, S_N is the sea quark distribution for quarks making up the dipole, x is the momentum fraction carried by a parton, and α_s is the running strong-interaction coupling constant. Note that Eq.(1) predicts a substantially more rapid increase of the dipole-hadron cross section with increase of energy $\propto x G_N(x, Q_{eff}^2) \propto x^{-n(Q_{eff}^2)}$, $n(4 \text{ GeV}^2) \sim 0.2$ than for the soft processes. This expectation is qualitatively different from the expectation of the two gluon exchange model where the cross section does not depend on energy [20]. CT for high energy scattering from nuclear targets was observed for coherent J/ψ production [21]. The experiment [22] performed at Fermi Lab with the 500 GeV pion beam confirmed the key CT predictions of Ref.[15]. In particular, the authors reported a strong increase of the cross section in the $\pi + A \rightarrow \text{“two jets”} + A$ process with A (A =carbon and platinum): $\sigma \propto A^{1.61 \pm 0.08}$ as compared to the prediction $\sigma \propto A^{1.54}$.

The prediction of increase of transparency with exclusive light vector meson production [23, 24] is consistent with indications of the FNAL E-665 [25] and HERMES [26] data on the ρ^0 leptonproduction (though these data were taken in the kinematics which did not exclude production of hadrons in the nucleus fragmentation region).

At intermediate energies observing CT (in the kinematics where CC holds) is complicated by the effects of quantum diffusion [12, 13, 27]. Even if PLCs of hadrons are involved in the collisions the space-time evolution leads to expansion of the wave packets as they move away from the interaction point, so that at a distance, l_{coh} which is referred to as the coherence length

$$l_{coh} \sim l_0 \text{ fm} \cdot p_h / \text{GeV}, \quad (2)$$

the packet expands to a normal hadronic size. Theoretical estimates give l_0 in the range $l_0 \sim (0.35 \div 0.8) \text{ fm}$, see a review in Ref.[12]. Here p_h is the hadron momentum in GeV. This leads to a strong reduction of the CT effect over a wide range of incident energies. In particular the estimates of Ref.[27] indicate that the effect of CT in say $A(p, 2p)$ reactions at $\theta_{c.m.} \sim 90^\circ$ is greatly reduced over a wide range of energies $\leq 20 \text{ GeV}$. A high resolution experiment of pion production recently reported evidence for the onset of CT [28] in the

process $eA \rightarrow e\pi^+A^*$. These experimental results agree well with predictions of Ref.[29] where the effects of CT were calculated using the quantum diffusion model with Eq.(2) and a value of $l_0 = 0.57$ fm. This confirms the small scale of l_{coh} (a larger l_{coh} would lead to a stronger CT effect for heavy nuclei) and indicates that it would be very difficult to determine the degree of squeezing of the hadronic configurations in the $2 \rightarrow 2$ processes at a wide range of momentum transfers using the $A(h, h'N)$ reactions. The difficulty is that understanding the dynamics are forces us to study two phenomena at the same time - squeezing at the initial point and the pattern of expansion.

Here we suggest a new strategy which allows the suppression of effects related to the space-time evolution of the wave packet and also allows checking the onset of the dominance of the contribution of PLC at moderate energies. This strategy uses novel hard branching $2 \rightarrow 3$ processes [8] (see Fig. 1)

$$a + b \rightarrow c + d + e. \quad (3)$$

The reaction is considered in the limit [8]:

$$-t' = -(p_b - p_d)^2 \rightarrow \infty, \quad s' = (p_c + p_d)^2 \rightarrow \infty, \quad \text{and} \quad -t'/s' \rightarrow \text{const}, \quad (4)$$

and

$$-t = -(p_a - p_e)^2 = \text{const} \leq m_N^2, \quad (5)$$

where p_i is the momentum of hadron i ($i = a, b, \dots, e$) and m_N is the nucleon mass. In this limit the leading-order QCD diagrams dominate the cross section for two-body processes [1]. As a result one can provide formal arguments [8] that the amplitude of the process is factorized into a product of the generalized parton distribution and the amplitude of large angle scattering of the projectile “ b ” off point-like $q\bar{q}$ or $3q$ configuration, see Fig.2.

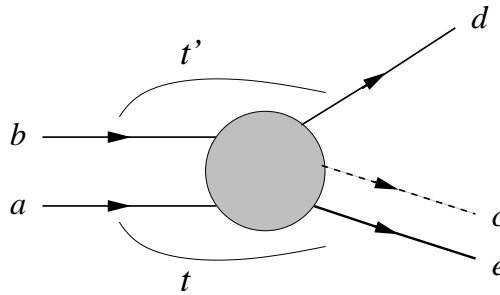


FIG. 1: $a + b \rightarrow c + d + e$ reaction.

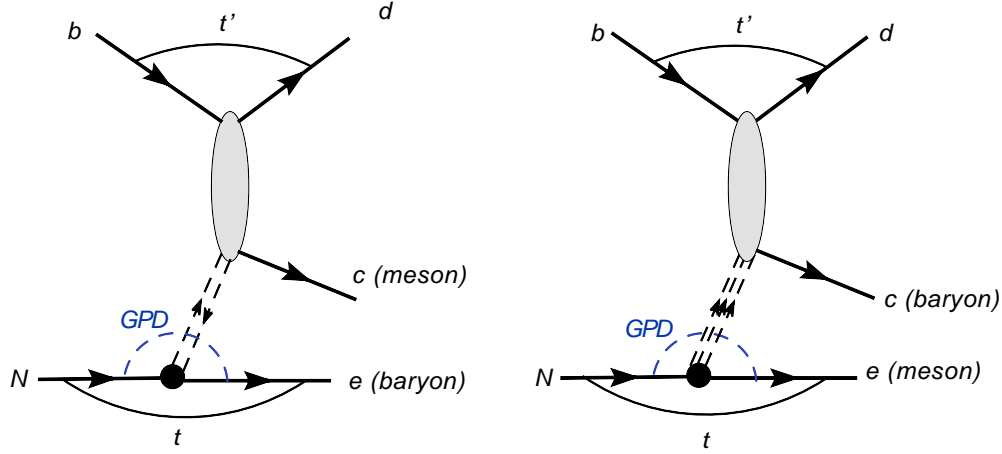


FIG. 2: (a) Production of fast meson and recoiling baryonic system. (b) Production of fast baryon and recoiling mesonic system.

In the case when b is the projectile, hadrons d and c carry practically all momentum of b , while the recoil particle carries small energy in the lab. frame:

$$\vec{p}_d = (xp_b, p_t^d), \quad \vec{p}_c = ((1-x)p_b, p_t^c), \quad p_t^d \approx -p_t^c \equiv p_t, \quad (6)$$

and

$$s' \simeq \frac{m_d^2 + p_t^2}{x} + \frac{m_c^2 + p_t^2}{1-x}, \quad -t \simeq \frac{(s' - m_b^2)^2}{s}. \quad (7)$$

In kinematics with $x \sim 1/2$, two leading particles carry large longitudinal momenta of about one half of the projectile momentum. This feature of the discussed process gives it a great advantage for investigating the dynamics of the large angle $2 \rightarrow 2$ processes using nuclear targets. Indeed, in contrast with the elementary $2 \rightarrow 2$ process, in the case of the $2 \rightarrow 2$ process embedded in the $2 \rightarrow 3$ process there is no correlation between momentum of the projectile and the value of p_t of the produced hadrons d, c . In the $2 \rightarrow 2$ process at a fixed scattering angle (*e.g.* $\theta_{c.m.} = 90^\circ$), there is one to one correspondence between the projectile momentum and the value of p_t . In a sense, using the $2 \rightarrow 3$ kinematics lets one boost hadrons in the PLCs relative to the nucleus, thereby practically completely freezing the PLCs. This is achieved in the limit when we keep p_t and mass of the c, d pair the same, hence preserving the kinematics of hard scattering but allowing the total momentum of the pair to vary. As a result, one can regulate the degree of freezing of the pair while it propagates.

As an example, let us consider a test of CT in elastic $\pi^+\pi^-$ scattering. We choose

this example for several reasons: Firstly, due to the minimal number of constituents in this process we expect a lower-energy onset of the CT regime than, say, in pp scattering. Secondly, the rate of the space-time evolution in this case is constrained by the pion electroproduction data [28]. Thirdly, it is known that the cross section of the elementary process

$$\pi^- p \rightarrow \pi^- \pi^+ n, \quad (8)$$

is sufficiently large as it was measured at FNAL at 100 GeV/c and 175 GeV/c including kinematics where $s', -t'$ are of the order of few GeV^2 [30]. The data indicate dominance of the pion exchange in t -channel which is consistent with the expectations for the hard kinematics as the pion-pole dominated GPDs (generalized parton distributions) give significantly larger contribution than the ρ -pole dominated GPDs, cf. analysis of the $2 \rightarrow 3$ process $NN \rightarrow N\pi N$ in Ref.[8]. Also, the COMPASS collaboration at CERN has collected large statistics for forward pion production for 190 GeV pion scattering off a wide range of nuclei [31] and has plans for further measurements using the pion beam. There are other interesting channels for measurements with pion and proton beams. We briefly discuss these channels at the end of this article.

Since coherent scattering for a nuclear target, is negligible for this reaction, the process we examine is $\pi^- A \rightarrow \pi^+ \pi^- A^*$, where the total energy of the residual system is close to $M_A - t/2m_N$. The underlying elementary process involves transition of proton to neutron, and it generates the final system of A nucleons with a small overlap with a nuclear-bound state. In principle, it may be difficult to experimentally exclude the production of an excited hadronic system, like the Δ -isobar (for example $\pi^- n \rightarrow \pi^- \pi^+ \Delta^-$). However the factorization theorem holds for any fixed mass of the produced system “ e ”. Typically, the experimental momentum resolution [31] for the detected pions is the same for different nuclei/hydrogen (^2H) targets. In this case, the cuts on the mass of the produced hadronic system (N, Δ, N^*, \dots) remain the same and would not affect our predictions for transparency.

First we consider CT effects for high energy projectiles - $E_\pi \sim 200$ GeV. In this case, Eq.(2) tells us that the coherence length of the final pions exceeds $30 \text{ fm} \gg R_A$, and is a factor of two larger for the projectile pion. Therefore expansion effects may be neglected. We define nuclear transparency as

$$T_A = \frac{\frac{d\sigma(\pi^- A \rightarrow \pi^- \pi^+ A^*)}{d\Omega}}{Z \frac{d\sigma(\pi^- p \rightarrow \pi^- \pi^+ n)}{d\Omega}}, \quad (9)$$

where Ω is the solid angle for the $\pi^-\pi^+$ system. This ratio can be estimated using the semi-classical approximation as [27]

$$T_A(\vec{p}_b, \vec{p}_c, \vec{p}_d) = \frac{1}{A} \int d^3r \rho_A(\vec{r}) P_b(\vec{p}_b, \vec{r}) P_c(\vec{p}_c, \vec{r}) P_d(\vec{p}_d, \vec{r}), \quad (10)$$

where $\vec{p}_b, \vec{p}_c, \vec{p}_d$ are three momenta of the incoming and outgoing particles b, c, d ; ρ_A is the nuclear density normalized to $\int \rho_A(\vec{r}) d^3r = A$ (for simplicity we neglect here a small difference between the proton and matter distributions). The probabilities of no interaction with the entering and 2 outgoing fast hadrons are given by the product of probabilities P_j

$$P_j(\vec{p}_j, \vec{r}) = \exp \left(- \int_{\text{path}} dz \sigma_{\text{eff}}(\vec{p}_j, z) \rho_A(z) \right). \quad (11)$$

Here P_j is the probability for particle j with momentum \vec{p}_j to propagate along a path from the point of hard interaction \vec{r} and z is the distance from the interaction point. Here, for generality, we write the expression allowing for expansion effects.

For soft interactions the effective cross section is given by $\sigma_{\text{eff}} \sim \sigma_{\text{tot}}(\pi N)$. As a result, in this limit Eqs.(10),(11) predicts values of transparency that drop strongly with increasing values of A , and which are dominated by the scattering off nucleons of the rim of the nucleus $\propto A^{1/3}$. For example, for the case of a soft interaction ($\sigma_{\text{eff}} = 25$ mb) $n(A) = \partial \ln(AT(A)) / \partial \ln A$ is about 0.30 (0.24) for $A=40$ (208). In the high energy limit, when the $q\bar{q}$ configurations of incoming and outgoing pions can be considered as frozen $T(A)$ can be estimated using Eqs.(10),(11) with z -independent σ_{eff} . For the purpose of obtaining rough estimates we will neglect a possible difference in the degree of squeezing of incident and outgoing pions as well as the energy dependence of the cross section as given by Eq.(1) (the second effect is definitely small as the gluon density changes in the discussed virtuality range less rapidly than $x^{0.2}$ which corresponds to the difference of the cross sections for initial and final pions of $2^{0.2} \sim 1.15$). We find that T_A is very sensitive to a variation of σ_{eff} - see Fig.3. The degree of squeezing can be estimated by considering the leading QCD diagrams for π - π scattering. For a pion with a given p_t , the internal characteristic momenta are $\sim p_t/2$, corresponding to the transverse size d of the dipole of the order $\frac{\pi/2}{p_t/2}$. For $p_t = 1.5$ (2) GeV/ c this corresponds to $d = 0.4$ (0.3) fm where Eq.(1) for a $q\bar{q}$ wave packet with energy 100 GeV gives $\sigma_{\text{eff}} \leq 4$ mb. Hence, if the perturbative mechanism of π - π scattering dominates in this p_t range, a large color transparency effect is predicted. For example, one can see from Fig.3 that for $A = 40$ (208), transparency $T(A)$ is predicted to increase by a factor ~ 4 (8) from

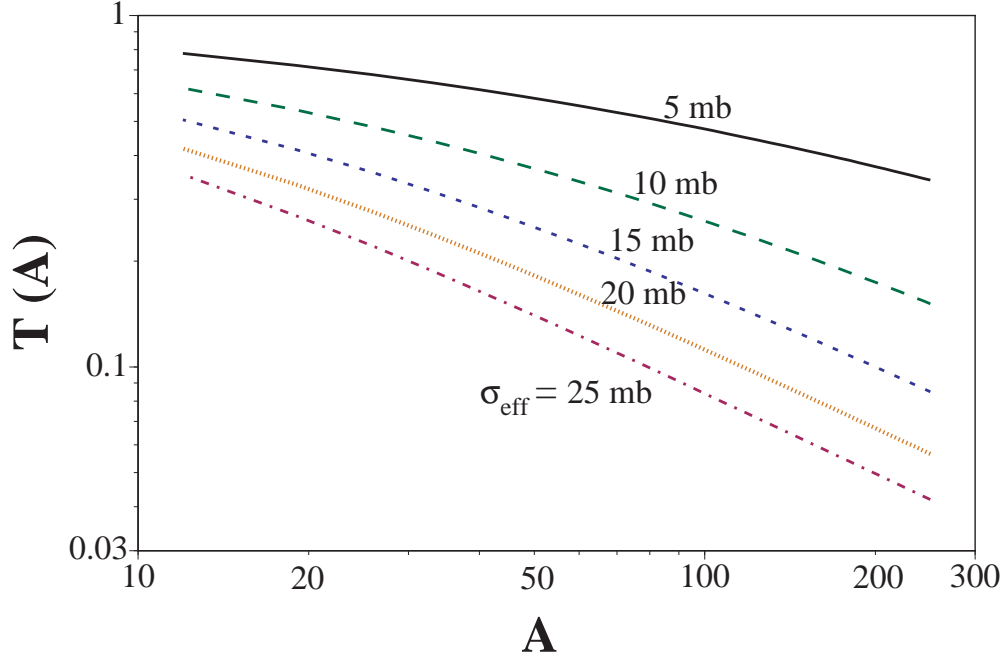


FIG. 3: A -dependence of $T(A)$ for different values of σ_{eff} .

its “Glauber-type” geometric value of $\sigma_{\text{eff}} = 25 \text{ mb}$ to $\sigma_{\text{eff}} = 5 \text{ mb}$. Since for small p_t one expects the geometric picture with $\sigma \sim \sigma_{\pi N}$ to describe $T(A)$ reasonably well, we expect that the ratio T_{A_1}/T_{A_2} should strongly depend on p_t . In the regime of small absorption one can determine σ_{eff} from a study of A -dependence and, by using Eq. (1), determine the average size of the color dipoles involved in the process.

Study of the space-time evolution of small wave packets

Assuming that the measurements at large energies observe the effects of CT, a next step would be to study $T(A)$ for production of the $\pi\pi$ pair for fixed s', t' as a function of the incident momentum, p_π . Indeed in this limit the $2 \rightarrow 3$ amplitude is factorized into the product of the amplitude describing the hard block of $2 \rightarrow 2$ process and the GPD describing coupling to the soft block. Therefore, the sizes of the hadronic configurations involved in the $2 \rightarrow 2$ large s', t' process should not depend on p_π at the interaction point. Hence the p_π dependence of $T(A)$ under these conditions should originate from the contraction of the small size configuration in the projectile b as it approaches the interaction point and expansion of the outgoing wave packages which evolve into hadrons c and d . At large values p_π contraction and expansion occur outside the nucleus (Fig.4a), while with decreasing values of p_π both contraction and evolution occur inside the nucleus (Fig. 4b). To estimate the

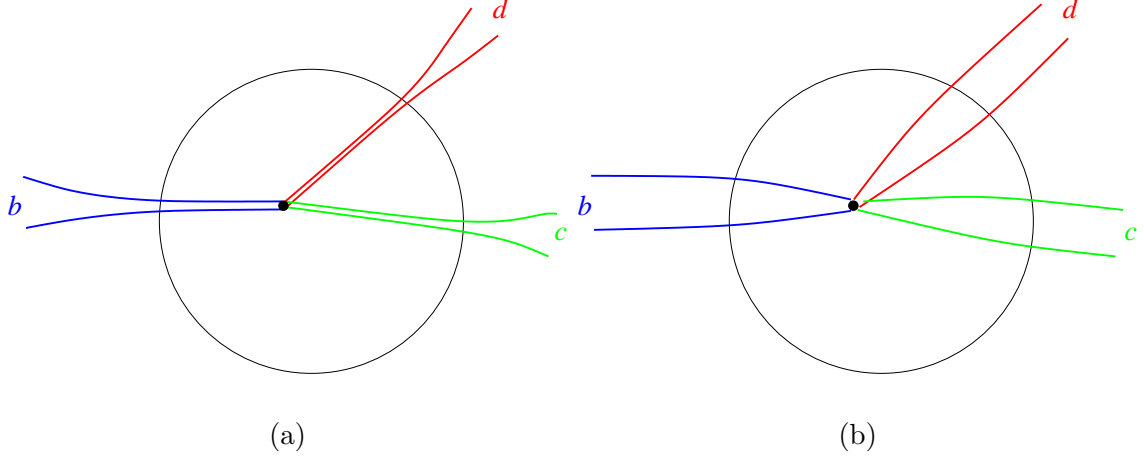


FIG. 4: Space-time evolution of the wave packets of incoming and outgoing fast hadrons for large (a) and moderate incident momenta (b).

expected effect we can use the quantum diffusion model of Ref.[27] which gives

$$\sigma_{eff}(z) = \left(\sigma_{hard} + \frac{z}{l_{coh}} [\sigma_{soft} - \sigma_{hard}] \right) \theta(l_{coh} - z) + \sigma_{soft} \theta(z - l_{coh}), \quad (12)$$

where z is the distance from the interaction point, l_{coh} is given by Eq.(2), σ_{hard} is the interaction of the PLC close to the interaction point and the interaction reaches the strength of soft interaction at $z = l_{coh}$, so $\sigma_{soft} \sim \sigma_{tot}(\pi N)$.

We performed numerical calculations using $l_0 = 0.57$ fm which gives a good description of the pion electroproduction data. The results of the calculation of $T(A)$ for symmetric configuration of two pions ($x \sim 0.5$) and $\sigma_{hard} = 5$ mb are presented in Fig.5 as a function of p_π . One can see from the figure that the optimal interval for study of the space-time evolution of the wave packets is $p_\pi = 20 \div 40$ GeV/c (we do not consider smaller p_π since in this case $|t_{min}|$ becomes too large) because $T(A)$ significantly changes in this p_π region. If CT is observed at high energies for sufficiently asymmetric configurations, say $x \sim 0.2$, a high precision study of $T(A)$ as a function of x may provide additional tests of the pattern of space-time evolution of the wave packages. In this case the main contribution is given by the expansion of the wave packet forming a slower pion. In the quantum diffusion model we find that stronger absorption of a slower pion is partially compensated for by a weaker absorption of the faster pions resulting in a relatively small overall change of the transparency in Fig.6.

In conclusion, a series of measurements for the same configuration of the two pion system (the same p_t and $M_{\pi\pi}$) corresponding to the same sizes of the PLCs in the interaction point for a range of p_{inc} would provide unique information about the space-time evolution of high

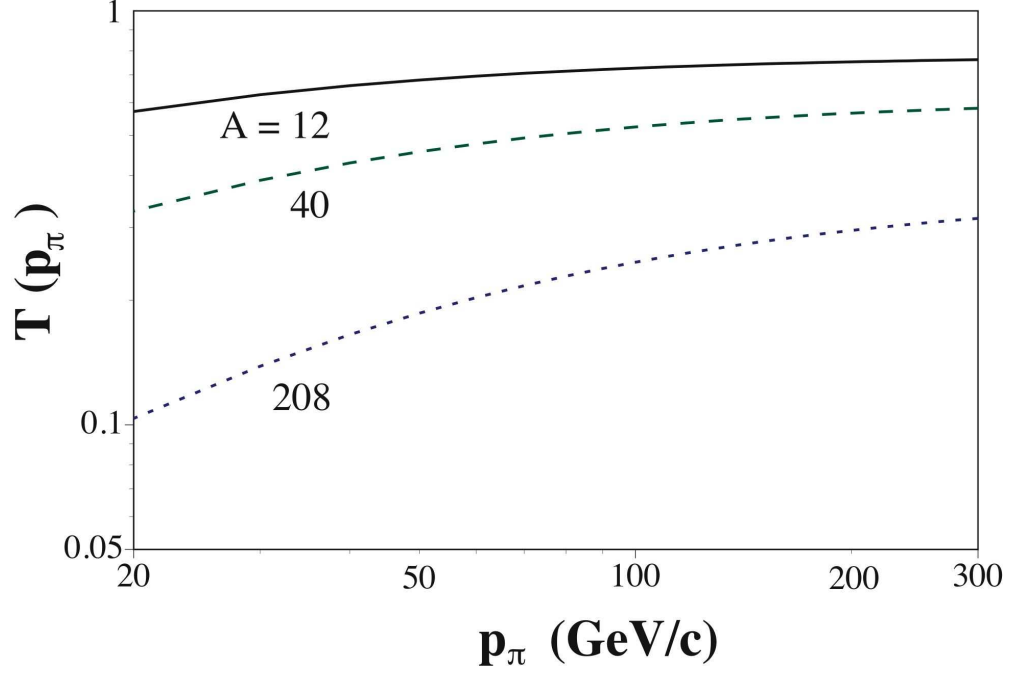


FIG. 5: p_π -dependence of $T(A)$ for different nuclei and $\sigma_{hard} = 5$ mb.

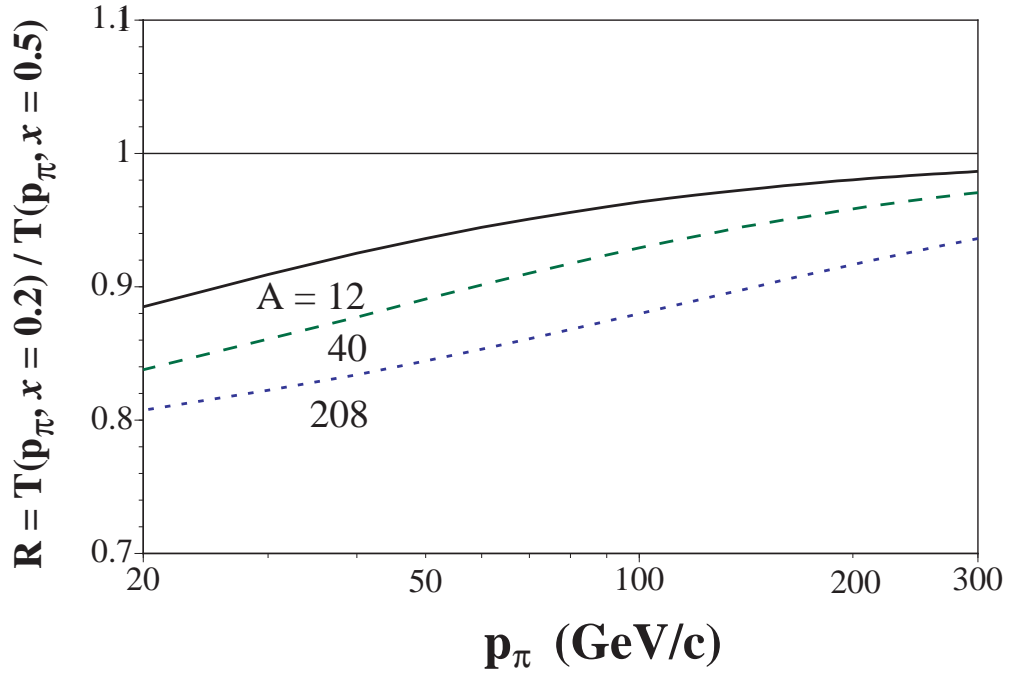


FIG. 6: p_π -dependence of the ratio $R = T(A)_{x=0.2}/T(A)_{x=0.5}$ for different nuclei.

energy wave packets.

Other channels

Above we focused on the $\pi^-\pi^+$ state channel. Obviously there are many other interesting channels. Here we give just few examples:

- For an isospin zero target the ratio of $2 \rightarrow 3$ cross sections for production of $\pi^-\pi^-$ and $\pi^-\pi^+$ in the factorization limit is equal to the ratio of the $\pi^-\pi^-$ and $\pi^-\pi^+$ elastic scattering. Based on the observation of the enhancement of the cross sections of the processes where quark exchange is allowed [32] one could expect that the ratio $\sigma_{el}(\pi^-\pi^-)/\sigma_{el}(\pi^-\pi^+)$ is much larger than one for $-t'/s' \sim 0.5$. Similarly one would be able to measure the ratio of the $\pi^-\pi^+$ and $\pi^-\pi^0$ elastic cross sections.

- One expects to have a significant rate of the process $\pi^+ + A \rightarrow \pi^+ + K^+ + A^*$ since the smallness of the GPDs describing nucleon to hyperon transitions as compared to the pion-pole dominated GPD is compensated to some extent by a larger probability for kaon than for pion to be in the PLC which is given by the factor $f_K^2/f_\pi^2 \approx 1.4$, where f_π and f_K are pion and kaon decay constants [33].

- One can use (anti)proton beams to study the onset of CT in πN scattering using production of the leading N and π with back to back large transverse momenta.

- One can use high energy proton beams to look for production of two back to back protons with the same invariant energies as the ones studied at BNL [5] to check whether oscillations of the transparency would be present for invariant energies of the proton pair matching BNL invariant energies.

In summary, we presented a new method for probing the dynamics of large angle hadron-hadron scattering using the CT phenomenon which is free from the limitations imposed by the expansion effects of the PLCs. It can be applied to a much broader range of two body processes than the original method including meson-meson scattering ($\pi\pi, \pi K, KK$) where one expects an earlier onset of the CT regime than for meson(baryon)-baryon scattering. One could also look for the onset of CT in the meson-baryon ($\pi N, \Lambda\pi, \dots$) and baryon-baryon ($pN, p\Delta, \dots$) scattering. Studies with beams of energies in the $20 \div 200$ GeV range appear to be optimal for these purposes [31, 34, 35]. If successful, such experiments will open the way to measuring GPDs of a wide variety of hadrons.

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